MODELING AND SIMULATION OF LAUNCH VEHICLE DIGITAL AUTO PILOT
Ashok Joshi, Professor, Department of Aerospace Engineering, Non-member, IIT Bombay, Mumbai, India
P G Pramodh, Scientist/Engineer PSLV Project, Non-member, VSSC, Thiruvananthapuram, India

Abstract
This study is concerned with the effect of non-linearities in the configuration design of Digital Auto Pilot (DAP) in launch vehicles. An electro-hydraulic actuator model of a launch vehicle control system is considered for analysis of non-linearities. Various non-linear effects like saturation (in current and stroke limit), dead zone and coulomb friction are taken into account. DAP, which is an interface between the guidance system and control system, is designed to cater to the model (linear/ non-linear) adopted for the actuator. In the actuator alone case, without considering the total flight regime and vehicle model, the performance is found to be satisfactory for linear as well as non-linear actuator models. In the actuator-vehicle combination, when the simulation is carried out for the total flight regime considering the vehicle model, the performance of the linear / non-linear actuator model is dependant on DAP configuration. This study brings out the fact that the DAP configuration is specific to the actuator model, so that satisfactory performance of launch vehicle control system can be ensured only by choosing proper configuration for DAP, based on consideration of non-linearities in actuator model.

Nomenclature

D Drag Force (N)
F_A Actuator Force (N)
F_N Force on the Engine/ Nozzle (N)
l_x Inertia of Vehicle about Swivel Point (kg.cm²)
l_Y Inertia of Vehicle about Pitch Axis (kg.cm²)
k_a Actuator Servo Amplifier Gain (mA/V)
k_C Flow Pressure Coeff. of Servo valve (cc/s/ksc)
k_L Equivalent Stiffness of Mounting (kgf/cm)
k_p LVDT Sensitivity of Servo valve (V/rad)
k_q Linear Equivalent Gain of Servo valve (cm²/s)
k_v Spool Valve Current by Displacement (cm/mA)
L Actuator Lever Arm (cm)
L_p1 Length of Slop Pendulum (m)
l_c Distance of Vehicle Nose-Body Axis Origin(m)
l_a Distance of Vehicle Nose-Body Axis Origin(m)
l_0 Reduced mass of Vehicle (Kg)
l_p Mass of Slosh Pendulum (Kg)
l_r Mass of Rocket engine (Kg)
P_f No load Pump Delivery Pressure (ksc)
P_s Load Pressure in Actuator (ksc)
P_s Fluid supply Pressure in Actuator (ksc)
Q_m Maximum Pump Delivery rate (cc/s)
Q_p Pump Flow (cc/s)
Q_c Control flow from Servo valve (cc/s).
T_C Control Thrust (N)
T_F Forward Thrust (Un-gimballed thrust) (N)
T_v Actuator Spool valve time constant (s)
X_s Actuator Piston movement (cm)
X_v Servo valve Orifice opening (cm²)
Y_v Spool Displacement (cm)
δ Engine/ Nozzle deflection angle (rad)
θ Pitch Angle (deg)
Γ_p1 Pendulum Angle for first Slop Pendulum(rad)
µ_c Control Moment coefficient (sec⁻¹)
µ_p1 Slosh Force coefficient (Oxidizer tank) (sec⁻²)
µ_r Rocket Engine Inertia-Vehicle Inertia ratio
µ_a Aerodynamic Force coefficient (sec⁻³)
ω_p1 Slosh Frequency (Oxidizer tank) (rad/s)
ξ_p1 Slosh Damping Ratio (Oxidizer tank)

Introduction
Launch vehicles require a control system to stabilize them as well as to give trajectory related commands. In modern launch vehicles, this task is performed by a Digital Auto Pilot or a DAP. In the design of DAP, it is necessary to represent all the physical effects as accurately as possible. However, in many cases, linearized models of various subsystems are used for the design process. Such an approach can work if there are large control margins available, but a more efficient and optimal control design needs the mathematical models which have greater fidelity. In general, such models are non-linear in nature and sometimes it is necessary to determine if a DAP designed for a linearized system will work with same fidelity, if the models are nonlinear. The present study explores this aspect of the DAP design in launch vehicles.
A literature survey has been carried out, regarding different types of controllers and controller design methods, currently in practice [8]. Various algorithms for the selection / design of controller / auto-pilot are also discussed. In addition, the types of non-linearities present in the launch vehicle models, particularly in actuator systems, are discussed in detail. Objective of the present study is to carry out modeling and simulations of electro hydraulic actuator used for nozzle angular positioning and assess its performance without and with non-linearities taken into account.

**Linear Modeling & Simulation of Actuator** [3, 8, 9]

The critical elements of the linear actuator model are servo valve, hydraulic pump, actuator chamber and piston and engine gimbal dynamics. Nozzle dynamics is also considered in the modeling. In this case, various non-linearities associated with the electro hydraulic actuator like saturation, dead zone, coulomb friction, preload, on-off hysteresis, backlash etc. are not considered. The following equations describe the mathematical model, which is shown graphically in fig. 1.

Servo Valve Dynamics:

\[ k_a, k_q, k_v / (1 + T_v s) = 1741/(1+0.002s) \]  \( \text{(1)} \)

- **Forward Filter Transfer Function:**
  \[
  0.25s^3 + 6.28s^2 + 369.3785s + 888.1250 \\
  s^7 + 101.1s^6 + 2589.8s^5 + 1579.4
  \]

- **Engine Gimbal Transfer Function:**
  \[
  73.32 \\
  (15910s^2 + 66667s)
  \]

- **Feedback Filter Transfer Function:**
  \[
  142122.3 \\
  (s^3 + 533.06s + 12122.3)
  \]

The numerical simulation is carried out with MATLAB®-SIMULINK® using fourth order Runge-Kutta algorithm with step size of 1 ms. Simulation results are analyzed for a duration of 1.5 s, with and without considering the nozzle/engine gimbal dynamics in the forward loop. Step response parameters like rise time, peak overshoot and settling time, are computed for both the cases and are found to match approximately with the physical case. (Figs. 2&3 and Table 1)

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**Fig. 1 Linear SIMULINK® Model of the Actuator**
Table 1: Comparison of Simulation Results and Design Specifications for Linear Actuator

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nozzle deflection (δₙ)</th>
<th>Actuator deflection (δₐ)</th>
<th>Specification (δₐ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise time (ms)</td>
<td>55</td>
<td>80</td>
<td>80 ± 20</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>33</td>
<td>19.5</td>
<td>20 ± 2</td>
</tr>
<tr>
<td>Settling time (ms)</td>
<td>680</td>
<td>260</td>
<td>250 ± 30</td>
</tr>
</tbody>
</table>

Fig 2. Linear Actuator Step Response

Fig 3. Linear Actuator Step response

Modeling of DAP for Linear Actuator \[^{10,11,12}\]

Digital Auto Pilot (DAP) for Linear Actuator consists of Attitude Filter, Forward Path Gain, Vehicle Transfer Function, Feedback Filter Transfer Function and Sensor Transfer Function. In the present case, an existing design of such a DAP is used for assessing the adequacy of the actuator modelling. The various transfer functions are as given below.

Attitude Filter Transfer Function:
\[
\frac{0.75 + 0.75 z^{-1}}{1 + 0.5 z^{-1}} \] \[^{11}\] (5)

Forward Path Gain, \( K_a = 0.85 \) \(^6\)

Forward Filter Transfer Function:
\[
\begin{align*}
0.241702 & + \\
\frac{1.3374555 z^{-1} + 1.4128671 z^{-2} +}{1 + 0.1517240 z^{-1} - 0.0586190 z^{-2}} & + \\
\frac{-1.314151 z^{-1} + 0.465605 z^{-2} +}{1 - 0.7496074 z^{-1} + 0.2760324 z^{-2}} & + \\
\frac{-0.1122301 z^{-1} + 0.1120510 z^{-2} +}{1 - 1.839706 z^{-1} + 0.840520 z^{-2}}
\end{align*}
\] \((7)\)

Vehicle Transfer Function:
\[
\frac{\mu_c}{s^2 - \mu_a} \quad \mu_c = 5.656, \mu_a = 0.0214 \] \(^8\)

Feedback Filter Transfer Function:
\[
\frac{0.75 + 0.75 z^{-1}}{1 + 0.5 z^{-1}} \] \(^9\)

Sensor Transfer Function:
- Position Gyro \[^{11}\]
  \[
  6168.5 / (s^2 + 109.955s + 6168.5) 
  \]
- Rate Gyro \[^{11}\]
  \[
  12090.265/(s^2+153.94s+12090.265) 
  \]

Figure 4 shows the schematic of the total system, which is simulated.
Linear Actuator Coupled with DAP

Numerical simulation of linear actuator with DAP is carried out using fourth order Runge-Kutta algorithm with step size of 1 ms. The following assumptions are made for the simulation.
1. Error in yaw and roll is neglected.
2. Rate control logic is not applied.
3. Gain scheduling is not considered.
Step response characteristics (Fig. 5) show a similar result, in comparison to that of actuator alone case. It is observed that the system is stable; whereas the settling time is observed to be higher.
Bode diagrams (fig. 6) also show a stable system.

Non Linear Modeling/Simulation of Actuator

The electro hydraulic actuator is modeled here as an actuator-nozzle combination. In the present modeling of the electro hydraulic actuator, few non-linear effects are considered, as in the actual system these effects also contribute to the system performance. In some cases, the accuracy of representation of physical dynamics is poor for linear models. Therefore, the consideration to the various non-linearities is expected to make the model response characteristics closer to the actual system specifications.
Servo Valve Non-linearity\(^{(9)}\)

Spool valve displacement \(Y_V\) directly controls the flow into the actuator chamber by varying the orifice opening, \(X_V\). Here the variation of \(X_V\) with respect \(Y_V\) is non linear. When the spool displacement is between a specified limit of \(\Delta_1\) and \(\Delta_2\) in forward or reverse direction, the variation is linear with one slope. When \(Y_V\) is between \(\Delta_2\) and \(Y_{V_{\text{max}}}\) slope in forward or reverse direction is different. Therefore when the total range of spool displacement is considered, the variation is non-linear with two slopes. The non-linearity can be incorporated in the model in the form of a look up table with seven data points as given below. It may be noted here that in the design of DAP, port opening \(X_v\) is considered as a state variable, and computed from this table.

<table>
<thead>
<tr>
<th>(X_v)</th>
<th>(Y_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0403</td>
<td>-0.0380</td>
</tr>
<tr>
<td>-0.0046</td>
<td>-0.0023</td>
</tr>
<tr>
<td>-0.0008</td>
<td>-0.0008</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0008</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.0046</td>
<td>0.0023</td>
</tr>
<tr>
<td>0.0403</td>
<td>0.0380</td>
</tr>
</tbody>
</table>

Hydraulic Flow Non-linearity\(^{(9)}\)

The control flow from the servo valve is given by,

\[
Q_v = K_N \cdot P_s \cdot \gamma \cdot P_L \cdot \text{sgn} (P_s - \gamma \cdot P_L) \\
\text{Where } \gamma = \text{sgn} X_v
\]

Here \(Q_v\) is dependant on \(P_s\) and \(P_L\). Supply pressure \(P_s\) is not constant and it varies depending on the pump delivery rate \(Q_p\). If the internal leakage of the pump is neglected, it can deliver a maximum flow rate of \(Q_m\) for a wide range of output pressures up to \(P_c\). When the flow demand is less than \(Q_m\), the pump output pressure increases proportionally up to \(P_0\). The pump supply pressure \(P_s\) for a delivery rate, \(Q_p < Q_m\) can be obtained as,

\[
P_s = P_0 - \left( \frac{(P_0 - P_c)}{Q_m} \right) Q_p \\
\text{Where } P_c = 205, P_0 = 212, Q_m = 360 & Q_p = P_L \cdot k
\]

In addition to the above, the following non-linearities are also considered.

- Saturation of servo valve current: \(\pm 10\) mA
- Saturation of actuator stroke: \(\pm 10.7\) cm
- Coulomb friction at actuator side: 0.256 Kgf

Modeling of Non-linear Actuator\(^{(7,8,9)}\)

Modeling is based on servo valve equations, hydraulic pump model, accumulator dynamics and force balance equations, which are as given below.

**Servo Valve Equations:**

\(Y_v(s) / i_v(s) = k_v / (1 + T_v s)\) \(^{(9)}\)

**Forward Loop Filter Transfer Function:**

\[
\frac{0.25}{(s+2.5)} \cdot \frac{(s^2+22.62s+1421)}{(s^2+100.5s+2527)}
\]

**Force Balance Equations:**

**Engine Gimbal Transfer Function** \(^{(13)}\):

\[
73.32 \frac{1}{15910s^2+66667s}
\]

**Feedback Filter Transfer Function:**

\[
142122.3 \frac{1}{s^2+533.06s+142122.3}
\]

Fig. 7 shows the schematic SIMULINK diagram of the non-linear actuator.
Fig 7 SIMULINK Model of Non-linear Electro-hydraulic Actuator (with nozzle dynamics)

Numerical simulations are carried out to verify the step response & frequency response characteristics. Actuator commands of 10%, 50% and 100% are used as inputs to the control power plant and the outputs are actuator deflection ($\delta_A$) and nozzle deflection ($\delta_N$).

The results of simulation, in terms of the step input responses, are presented in figures 8 to 13.

Fig 8 - Step Response for 10% Command

Fig 9. - Step Response for 50% Command
Fig 10 - Step Response for 100% Command

Fig 11 – Step Response for 10% Command

Fig 12 - Step Response for 50% Command

Fig 13 - Step Response for 100% Command

Table 3 Time Response of Non-linear Actuator

<table>
<thead>
<tr>
<th>Parameters</th>
<th>10%</th>
<th>50%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With Nozzle Dynamics</td>
<td>Without Nozzle Dynamics</td>
<td>With Nozzle Dynamics</td>
</tr>
<tr>
<td>Rise time (ms)</td>
<td>115</td>
<td>115</td>
<td>120</td>
</tr>
<tr>
<td>% Overshoot</td>
<td>13.72</td>
<td>35.1</td>
<td>34.3</td>
</tr>
<tr>
<td>Settling time (ms)</td>
<td>830</td>
<td>950</td>
<td>220</td>
</tr>
</tbody>
</table>
Non-linear Actuator Coupled with DAP

Here, the non-linear actuator model is coupled with present DAP designed using classical approach for a linear actuator, and the same is modeled in SIMULINK®. (See Fig. 14)

![SIMULINK Model of Non-linear actuator coupled with DAP Designed for Linear Actuator](image)

**Fig 14** SIMULINK Model of Non-linear actuator coupled with DAP Designed for Linear Actuator

An autopilot designed for a linear actuator will give satisfactory performance with a linear actuator only. When this is coupled with a non-linear actuator, the step response characteristic shows unstable behavior. Here the DAP used is being designed for a linear actuator, with out considering the non-linearities. (See Fig. 15)

![Step response](image)

**Fig 15** - Step Response for 50% command

### DAP Design Using LQR Approach

Here the design of the DAP for the pitch loop of a typical launch vehicle is carried out using Linear Quadratic Regulator approach. The non-linearities are treated as operating points, for the purpose of design. Slosh effects in both the tanks (fuel and oxidizer) are considered while, vehicle flexibility and engine inertia effects are neglected. Dead zone, saturation and servo valve non-linearity are incorporated in the model in a look up table format.

**Mathematical modeling**

Mathematical model of the launch vehicle is formulated from the force equation, moment equation and slosh equation.

**Force equation:**

$$F_z = m_p a_x \Gamma_p + m_a \delta_c + T_c \delta - (T_F - D) \theta - N_a$$  \hspace{1cm} (17)

**Moment equation:**

$$I_{XY} \dot{\theta} = T_c l_\gamma \delta + N_a l_\alpha \alpha - m_p l_a \Gamma_p + (I_R + m_l l_c) \delta$$  \hspace{1cm} (18)
Slosh Equation:

\[\Gamma_{p1} + 2 \xi_p \omega_p \Gamma_{p1} + \omega_p^2 \Gamma_{p1} = \]

\[\frac{1}{L_{p1}} \left( \sum F_i + (l_{p1} - L_{p1}) \theta \right) + \]

\[\frac{1}{L_{p1} m_0} \left( m_{p1} a_x \Gamma_{p1} - m_{p2} a_x \Gamma_{p2} - m_r l_r \omega_l^2 \delta_l + \right.\]

\[2 m_r l_r \xi_d \omega_l \delta_N - \left( T_r + m_r l_r \omega_l^2 \right) \delta_N + \]

\[(l_{p1} - L_{p1}) \left[ (\mu_c - \mu_e \omega_e^2) \delta_N + \mu_e \theta - \mu_{p1} \Gamma_{p1} \right] - \]

\[\mu_{p2} \Gamma_{p2} + \mu_e \omega_e^2 \delta_l - 2 \xi_d \omega_l \mu_e \delta_N \]

(19)

Modeling of the hydraulic actuator is based on the hydraulic flow equations and the force balance equations, which are,

\[X_v = -512 X_A - 500 X_r + 3755 \delta_b \]  \hspace{1cm} (20)

\[\delta_N = -4.19 \delta_N - 3106.67 \delta_N + 42.42 X_A \]  \hspace{1cm} (21)

\[X_A = 0.0053 X_A + 2.63 X_A - 192.83 \delta_N \]  \hspace{1cm} (22)

The above mathematical model of the launch vehicle, along with electro-hydraulic actuator, is converted to the standard state space format, using the standard procedures.

\[
\frac{dx}{dt} = Ax + Bu \\
y = Cx
\]

(23)

The state vector is defined as,

\[
\{ x \} = \{ \theta, \theta, \Gamma_{p1},\Gamma_{p2},\Gamma_{r1},\Gamma_{r2},\delta_N,\delta_N, X_A, X_V \}^T
\]

(24)

and \( \delta_b \) is taken as the control input vector. Further, both \( \theta \) and \( d\theta/dt \) are considered as the outputs from the system. For the present design, the system is found to be nonlinear as well as time varying, due to propellant consumption and therefore, five different time instants, 0 s, 40 s, 80 s, 120 s, 150 s are considered for obtaining the vehicle parameters from the data documents. Since, nonlinear effects like saturation, dead zone and servo valve non-linearity are also considered, state variables \( X_A \) and \( X_V \) show nonlinear behaviour.

For each of these time instant, the coefficients of matrix \( A \) and row vector \( B \), are computed for 10%, 50% and 100% command input.

**Optimal Gain Matrix Computation**

Optimal gain matrix for each set of \( A \) and \( B \) matrices is worked out using the LQR technique. In the present case \( Q \) and \( R \) are selected as a diagonal matrix having the elements as reciprocal of the square of the maximum specified values for the state vector variables and control variable respectively. If any of the state or control variable value is obtained as more than the allowable specification, all the elements are normalized with respect to this large value, in order to satisfy the constrains. This is repeated until the specifications are satisfied and the final \( Q \) and \( R \) matrices are obtained, from which the gain vector \( K \) can be computed from the MATLAB control tool box function, ‘LQRD’. The above steps are repeated for all the five different time instants and the optimal gain vectors are computed for them.

**Minimal State Observer Design**

In the present case all the state variables are not available for feedback. Only the state variables \( \theta \) and \( d\theta/dt \) are sensed in the present system using position and rate gyros and thus we need to estimate the rest of the state variables. This is done using the concept of minimum state observer. The state observer estimates the state variables based on the measurements of the sensed output and control variables. The estimator design is done in MATLAB using Ackerman’s formulae. The state vector is partitioned into two parts \( x_a \) and \( x_b \). The state variable \( x_a \) is equal to the output \( y \), which can be directly measured and \( x_b \) is the measurable portion of the state vector, as shown below.

\[
\begin{bmatrix}
x_a \\
x_b
\end{bmatrix} =
\begin{bmatrix}
A_{aa} & A_{ab} \\
A_{ba} & A_{bb}
\end{bmatrix}
\begin{bmatrix}
x_a \\
x_b
\end{bmatrix} +
\begin{bmatrix}
B_a \\
B_b
\end{bmatrix} \{ u \}
\]

\[ y = [1 \ 0] \begin{bmatrix} x_a \\ x_b \end{bmatrix} \]  \hspace{1cm} (25)

The eigen-values used in the Ackerman’s formulae are selected as \(-1\) for all the nine states that need to be estimated. Gain matrix is of order 9×2, and for both the columns same gain vectors are assumed.
A new DAP is designed (see figure 16), using the gain matrices generated from the above analysis and it can be seen that all non-linearities are included in the model.

**New DAP Coupled with Non-linear Actuator**

Non-linear model of the electro hydraulic actuator considered earlier is coupled with the DAP designed using the LQR approach. Simulation of the total system is carried out in MATLAB©-SIMULINK©. Numerical simulation is carried out using fourth order Runge-Kutta algorithm with step size of 1 ms. Step response and frequency response characteristics show a result similar to that of actuator alone case. In the step response it is observed that the system is stable and the characteristics are close to the specifications. Step response characteristics like rise time, percentage over shoot and settling time are observed as increasing with command percentage as expected. Settling time obtained for 100% command is slightly above the specification. Frequency response characteristics of the system also show that the performance is satisfactory and response is very close to the specifications. (See Figs. 17-20).
Conclusions

This paper brings out the dependence of the design of a typical launch vehicle Digital Auto Pilot (DAP) on the type of modeling adopted for the control actuator. Depending on linear / nonlinear model adopted for the actuator, DAP design needs to be different. In the actuator alone case both linear as well as non-linear actuators give satisfactory results. When coupled with DAP designed for linear actuator the simulation results for linear actuator are found to be stable. When the non-linear model of the actuator is coupled with DAP designed for linear actuator, the simulation results show unacceptable response. The new design of DAP is carried out using LQR technique in MATLAB. The performance of the nonlinear actuator, when coupled with the new design of DAP, is verified in the simulation and response parameters are found to match the specifications. It is concluded that, the DAP designed for linear actuator becomes inadequate for non-linear actuator model and satisfactory performance is possible only by a new design of DAP.

References

7. Sam. K. Zachariah, L40 EGC system mathematical model in Actuator–Nozzle configuration based on Q-series hardware test results., Internal technical report, VSSC/CASD/L40/14/99.


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