Configuration Design of a Generic Air-Breathing Aerospace Vehicle Considering Fidelity Uncertainty

J. Umakant*
Defence Research and Development Laboratory, Hyderabad-500058, India

K. Sudhakar†, P.M. Mujumdar†
Department of Aerospace Engineering, IIT Bombay, Mumbai-40076, India

and

S. Panneerselvam‡
Defence Research and Development Laboratory, Hyderabad-500058, India

Design of complex aerospace vehicles is inherently multidisciplinary in nature. For new class of vehicles, the design problem is further complicated due to non-availability of disciplinary analysis tools with sufficient fidelity. A multidisciplinary design process has been set up to accomplish configuration design of a generic air-breathing hypersonic vehicle. A Probabilistic technique is used to represent fidelity uncertainty with respect to a disciplinary metric and its effect is propagated onto a system level metric through Monte-Carlo simulation. A design that maximizes this system level metric is sought through optimization.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>Axial Force</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DACE</td>
<td>Design and Analysis of Computer Experiments</td>
</tr>
<tr>
<td>H</td>
<td>Overall height of vehicle</td>
</tr>
<tr>
<td>H_cr</td>
<td>Cruise altitude</td>
</tr>
<tr>
<td>L</td>
<td>Overall length of vehicle</td>
</tr>
<tr>
<td>m_a</td>
<td>mass flow capture of air</td>
</tr>
<tr>
<td>M_I</td>
<td>Mach number at intake entry</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>Th_deliv</td>
<td>Thrust deliverable</td>
</tr>
<tr>
<td>TOGW</td>
<td>Take-Off Gross Weight</td>
</tr>
<tr>
<td>t_pl</td>
<td>Tail planform factor</td>
</tr>
<tr>
<td>w_pl</td>
<td>Wing planform factor</td>
</tr>
<tr>
<td>( \theta_1 ), ( \theta_2 ), ( \theta_3 )</td>
<td>Fore-body compression angles</td>
</tr>
<tr>
<td>( \delta_{\text{trim}} )</td>
<td>Control deflection for trim</td>
</tr>
<tr>
<td>( \theta_w )</td>
<td>Wing cant angle</td>
</tr>
</tbody>
</table>

I. Introduction

Air-breathing aerospace vehicles flying at hypersonic Mach numbers and employing scramjet propulsion, have a strongly coupled airframe and engine configuration. Typically, the entire undersurface of such vehicles can be
considered to be part of the propulsion system resulting in interdisciplinary couplings. In order to realize the performance potential of the configuration it is imperative to consider such interdisciplinary couplings with sufficient fidelity in the preliminary design phase itself. However, lack of a well established multidisciplinary design methodology and disciplinary analysis tools introduce uncertainties in the design environment. Thus there is a need for a design approach that enables handling of interdisciplinary coupling together with the capability to address the effect of various uncertainties on the configuration design. Initial studies at DRDL, Hyderabad\(^1\) focused on using traditional parametric and trade-off techniques for flow path design of a generic hypersonic vehicle. Subsequently the authors of this paper developed a multidisciplinary design tool to consider the effect of interdisciplinary coupling on a system level performance metric. Configuration design through formal optimization techniques was demonstrated\(^2\) using engineering methods to represent aerodynamics, propulsion, sizing and trajectory disciplines. Bowcutt\(^3\) has reported similar work earlier in 2001.

The present study focuses on improving the design process in Reference 2 through the use of probabilistic methods. Specifically we seek to propagate fidelity uncertainty associated with a discipline level metric on a system level metric through Monte-Carlo simulations. Various approaches have been advocated to consider the effect of uncertainty in a design environment. Mantis and Mavris\(^4\) have shown a Bayesian approach to non-deterministic Hypersonic Vehicle design. DeLaurentis\(^5\) created meta-model for cumulative distribution of the objective function and combined it with a probabilistic design technique to achieve robust design solutions. Another way of ensuring robustness in design has been demonstrated by Olds\(^6\) through application of Taguchi techniques for the design of Single-Stage-To-Orbit vehicle.

II. Design Problem Statement

It is required to design a generic aerospace vehicle cruising at hypersonic Mach number, using scramjet propulsion. Figure of merit to assess various designs is the thrust minus drag margin. The design constraints on the configuration optimization problem are as follows:

i) Dry weight of the vehicle to be within a prescribed maximum.

ii) Wing planform and tail planform are constrained from folding considerations.

Two constraints are included to impose realistic input conditions for intake and combustor design.

iii) Decelerate the free-stream Mach number to allowable values at the intake entry.

iv) Static pressure at the intake to be above a threshold value.

A constraint that accounts for actuator power limit through allowable controls deflection is imposed. This also helps in restricting the angle of attack of operation.

v) Control deflection required for trim flight conditions to be within allowable values

vi) Cruise altitude, a mission related constraint, to be within prescribed bounds

Thrust deliverable is the maximum available thrust and the drag experienced by the vehicle has to be less than this value. A configuration that meets all the above constraints and also maximizes the objective function, namely thrust minus drag margin, is sought.

III. Parameterization and Trade-Offs

A flow chart for configuration design is shown in Figure 1. The subsystem analysis modules that are required have been developed and are also shown in the same figure. Multi-Disciplinary Feasible analysis is performed at each step as the optimizer explores the design space for an optimum design. A meta-model for thrust deliverable is used wherein the effect of fidelity uncertainty in mass flow of air is captured using a probabilistic approach. Parameterization of the vehicle configuration forms the first step for implementing an optimization based design process. Figure 2 shows a sketch of the configuration, indicating the parameters. In the present study, nine independent variables have been identified to define the vehicle geometry. These are the three fore-body compression angles, the nose planform angle, mid-body length, the after-body length and expansion angle, the wing cant angle, and scale factors to control the wing and tail planform areas. Design variables to be optimized are a subset of these parameters. Each of the selected variables significantly affects the performance characteristics of the vehicle through more than one discipline. Table 1 lists performance parameters that each of these variable influences.

IV. Analysis Models

A. External Configuration Model

This model provides definition of the vehicle and sizing properties like mass, center of gravity and internal volume available. The nine independent geometric variables described in the previous section are the input for this

American Institute of Aeronautics and Astronautics
model. The body cross-section is grossly rectangular with faired corners. The upper surface is elliptical having a specified elliptic ratio. Using the configuration properties described above, analytical expressions are developed for estimating the internal body volume, wetted area and centers of gravity. Scaling the respective non-dimensional locations of other components like battery, actuator with respect to the baseline configuration derives the centers of gravity of these components. A surface area density correlation factor based on baseline configuration studies is used to estimate the weight of the airframe. Planform area factors returned from the optimizer, scale the wing and tail geometries and its properties with reference to the baseline configuration. The overall take-off gross weight and center of gravity is then obtained by appropriately summing up the individual properties.

B. Aerodynamic Model

The aerodynamic characteristics of the body are evaluated using tangent wedge/cone method and the wing and tail characteristics are evaluated using tangent wedge method. The definition obtained from the External Configuration Model, is used as input. In addition, Euler CFD studies have been carried out on the baseline configuration for calibrating the engineering models. Table 2 highlights the comparison of the normal force, pitching moment and axial force characteristics obtained using the engineering method and Euler calculations. In the present study, these calibration factors are assumed to be invariant in the design space.

C. Trim Model

This model estimates the angle of attack and control deflection required for trim using cruise conditions of lift equals weight and thrust equals drag. Mass and center of gravity from the External Configuration Model and the aerodynamic characteristics from the Aerodynamic Model, constitute the inputs. In addition to the aerodynamic normal force and pitching moment, the propulsive normal force and propulsive pitching moment contributions to trim are also accounted by representing the thrust as integral components in the aft-body and mid-body. The effectiveness of the tail surface is checked and if the trim control deflection required exceeds the allowable limit, additional ballast weight is used to achieve trim.

D. External Compression

Assuming shock on lip condition and using oblique shock theory, this model evaluates the fore-body ramp lengths and flow properties at the intake entry plane. The fore-body angles constitute input. Mass flow capture of air is computed for a given cross-sectional size of the intake. However, typical comparison with CFD generated results, shown in Table 3, reveal that the captured mass flow is over-predicted by about 20%. The error can be attributed to the fact that the engineering model assumes two-dimensional flow whereas CFD result, shown in Figure 3, clearly reveals the three-dimensional nature of the flow.

E. Thrust Model

This model is used to check if the vehicle drag is less than the maximum thrust deliverable. For the air mass flow captured, obtained from the External Compression Model, fuel flow rate is estimated at equivalence ratio of one. A look-up table of specific impulse for a hydrocarbon fuel has been generated as a function of Mach number and altitude. Thrust deliverable for cruise condition is then estimated using specific impulse from the look up table and the fuel flow rate. As a part of further research, authors of this paper have initiated work to create an engine deck through parameterization of intake-combustor-nozzle and using CFD to generate a database. In the present model structure, one of the factors that affect the accuracy of thrust deliverable is the fidelity uncertainty associated with mass flow capture of air. This is discussed further in section V.

V. Fidelity Uncertainty Modeling

For new class of vehicles, such as the one considered in this study, disciplinary knowledge and modeling capability is not adequate and well established. Detailed experimental studies are also not feasible in the conceptual design phase. In fact, high fidelity information on disciplinary metrics may exist only at a few design points. For the purposes of our study, we assume that very few CFD simulations, representing high fidelity observations, are available for a disciplinary metric. This necessitates the use of engineering methods, albeit with low fidelity, to explore the entire design space leading to the presence of fidelity uncertainty in the design process.

A. Approach

Step1: Based on the few available high fidelity observations, correlation factors (ratio of high fidelity observation to low fidelity observation) are generated for the disciplinary metric, namely captured mass flow of air. The variations
in the correlation factor are represented as a PDF that serves as an Input Uncertainty Model. Mantis\textsuperscript{4} had earlier used such an approach.

Step 2: Pre-determined design points are selected using Design of Experiments theory. At each of the sampled points, Monte-Carlo simulation is carried out to propagate the uncertainty in captured air mass flow estimations, resulting in a CDF for the system level metric, namely thrust deliverable.

Step 3: The CDF’s are then used to set up a database of the system level metric corresponding to desired reliability level. A meta-model for the system level metric, embedding the effect of fidelity uncertainty, is used in design optimization to determine the optimum settings of the design variables.

Step 4: This involves comparison of the solution obtained at step 3 with alternative methods. We perform the design optimization using engineering method in a deterministic manner i.e., we do not consider the effect of fidelity uncertainty. Next we assume that sufficient numbers of high fidelity simulations of mass flow capture of air are now available. A meta-model is created to represent the high fidelity simulations. Optimization is carried out using this meta-model, thereby mitigating the effect of disciplinary fidelity uncertainty. The variation in performance of the objective function is examined with respect to that obtained in step 3.

B. Meta-Modeling

Meta-modeling is done to create responses for the probabilistic based system level metric and the high fidelity based disciplinary metric. Sacks\textsuperscript{8} et al have argued that when modeling a deterministic computer code, it is more appropriate to use DACE or Kriging, rather than regression based techniques, for creating the response surface. This provides motivation to use DACE. Space filling designs, like Latin hypercube\textsuperscript{8}, are usually adopted to select the design sites at which the computationally expensive function needs to be evaluated for creating a Kriging model. The number of sample points needed depends on the number of design variables. Simpson\textsuperscript{8} has created a Kriging model using 25 sample points, based on L2 orthogonal experimental array, for a 3 variable aerospike nozzle problem. Keane\textsuperscript{10} has used 250 points of an LP\textsubscript{τ} array to generate a Kriging model of drag coefficient, for wing optimization studies with 11 design variables.

In this study the expensive function, namely mass flow of air capture, depends on the three fore-body compression angles. Design of Experiments (DoE) is performed using Latin hypercube sampling available in MATLAB toolbox.\textsuperscript{11} Latin hypercube sampling (LHS) is a strategy for generating random sample points ensuring that all portions of the design space are represented. Two levels were chosen in each direction and thus the original three-dimensional design space was divided into 8 sub-regions. Using LHS strategy, 40 points were selected in each sub-region and then 4 points were randomly picked from this set. This gave 32 sample points filling the entire design space. DACE toolbox in MATLAB is used for Kriging. For high fidelity simulations at the pre-determined sample points, Euler CFD code PARallel Aerodynamic Simulator ‘PARAS’\textsuperscript{12} is used. ‘PARAS’ is a CFD code developed at VSSC, Trivandrum, and is capable of simulating three-dimensional fluid flow past arbitrary shaped bodies.

C. Application

In the present study it is assumed that initially only four high fidelity observations, highlighted in Table 3, are available for the disciplinary metric. Low fidelity estimations of the disciplinary metric are made at these points to generate correlation factors. These are also highlighted in Table 3. Since the correlation factors are all one-sided, Weibull probability distribution, given below, is used to represent the fidelity uncertainty.

\[
f(t) = \frac{\beta}{\alpha} \left(\frac{t - \theta}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t - \theta}{\alpha}\right)^{\beta}\right] ; \ t \geq \theta
\]

\(\beta, \alpha\) and \(\theta\) are respectively the shape, scale and location parameters of the distribution. The Cumulative distribution function is given as

\[
F(x) = \int_{\theta}^{x} f(t)dt = 1 - \exp\left[-\left(\frac{x - \theta}{\alpha}\right)^{\beta}\right]
\]

and 1-\(F(x)\) represents the reliability function.
This can be re-written as

\[
W(X) = \beta X + A \quad \text{where}
\]

\[
W(X) = \ln(-\ln(1-F(x)))
\]

\[
X = \ln(x-\theta)
\]

\[
A = -\beta \ln(\alpha)
\]

In the relation for \(W(X)\), substituting the correlation factors for \(x\) and assuming the location parameter \(\theta\), the remaining parameters \(\beta\) and \(\alpha\) are estimated through linear regression. Kolmogrov-Smirnov test is used to check the goodness of fit. Since, in this study, the correlation factors are always greater than one, \(\theta\) is assumed to be one. Estimates of the probability distribution parameters \(\beta\) and \(\alpha\) quantify the fidelity uncertainty with regard to the disciplinary metric. The Input Uncertainty Model described in the previous section is thus defined and is shown in Figure 4. For Monte-Carlo simulations, one thousand points are randomly selected from the Input Uncertainty Model. A histogram of these points is shown in Figure 5. System level metric is evaluated at these points to yield its probability distribution. This exercise is repeated for each of the 32 DoE selected points. The histogram, cumulative distribution function and reliability function of the system level metric, for a typical case, is respectively shown in Figures 6, 7 and 8. In this study we have chosen to select from the cumulative distribution function, system level metric corresponding to 95% reliability. Thus a database of 32 design points at which the system metric, namely thrust deliverable, corresponding to 95% reliability is now set up. This database is used to create a DACE meta-model for the system level metric. The DACE fit consists of a regression part represented by a constant and a stochastic part represented by Gauss function. Details on DACE method are described in Reference 8. Cross-Validation diagnostic plot, shown in Figure 9, is used to assess the accuracy of the fit. Since all the points are very close to the diagonal, the fit is considered to be accurate. Similarly a DACE fit for high fidelity simulations of the disciplinary metric, needed in step4 of the previous section, is also constructed. In this case the fit has a regression part consisting of a quadratic polynomial and a stochastic part consisting of a Gauss function.

VI. Configuration Design Optimization

The configuration design problem is formulated as an optimization problem. The design process formulation, shown in Figure 1, is referred to as ‘Multi-Discipline Feasible’ formulation where the optimizer sees only converged solutions of multi-disciplinary analysis. A Sequential Quadratic Programming based optimizer ‘FFSQP’ is used. Gradients are calculated using forward finite difference approximation. Optimization is carried out to determine the settings for the three fore-body compression angles, wing cant angle, planform area scale factors for wing and tail and cruise altitude, that maximize the margin of thrust deliverable minus drag. All the designs are constrained to have the same body width and fuel weight. The problem statement and optimization constraints are highlighted in Table 4. The constraints are on the allowable overall length and height of the vehicle, take-off gross weight, intake entry Mach number, and control surface deflection. Side constraints on fore-body compression angles are imposed such that fore-body length and height are allowed to vary within 15%. Wing cant angle is allowed to vary from 0° to 6°. Scaling of the wing planform is restricted to within 20%, while 30% variation is allowed on the tail planform factor. Variation in cruise altitude is within five kM. All the variables are coded to range from zero to one and the constraints are also normalized.

Three optimization design exercises were performed for the same problem statement, highlighted in Table 4. Design-Exercise 1 uses low fidelity methods for evaluation of all the constraints and the objective function. The solution obtained is deterministic wherein the fidelity uncertainty is not considered. In Design-Exercise 2, the objective function is evaluated using the DACE meta-model for 95% reliable thrust-deliverable wherein the effect of input fidelity uncertainty is embedded. Design-Exercise 3 uses high fidelity based DACE meta-model for the evaluation of disciplinary metric, mass flow capture of air. This is assumed to eliminate the fidelity uncertainty effect of the disciplinary metric on the evaluation of the objective function. The solutions obtained in Design-Exercises 1 and 2 are compared with that obtained in Design-Exercise 3 to assess the effectiveness of the probability based design methodology. In addition, another high fidelity simulation for the disciplinary metric is carried out at the design variable settings of Design-Exercise 2 and the system level metric is computed again with this input and compared with probability based objective function.

The results from the three exercises are shown in Table 5. It can be observed that the side constraints on wing cant angle and wing planform scale factor are active at their respective upper bounds. Further increase in the wing cant angle will increase the local angle of attack and may result in aerodynamic heating problem, while sizing
considerations restrict further increase in wing planform area. It can be observed that the objective function from Design-Exercise 1 is about 33% higher than the high fidelity based solution. However, when the effect of fidelity uncertainty is considered in Design-Exercise 2 the objective function is about 14% lower compared to the high fidelity based solution from Design-Exercise 3. Thus though the low fidelity based design processes offer ease of use and low computational overhead, their accuracies are not acceptable to enable design decisions to be taken even in the preliminary design phase. On the other hand combining the low-fidelity based design method with probabilistic techniques to represent fidelity uncertainty greatly improved the comparison with the high fidelity based solution, enabling better design decisions. The system level metric result of Design-Exercise 2 was further verified by performing an additional high fidelity simulation for the disciplinary metric at the corresponding design variable setting. The objective function thus computed is within 5% compared to that obtained as part of Design-Exercise 2.

VII. Conclusions
A design methodology that allows consideration of disciplinary fidelity uncertainty and includes the effect of interdisciplinary couplings is showcased with respect to configuration design of a generic air-breathing aerospace vehicle. The results demonstrate that the accuracy of a low-fidelity based design process is greatly enhanced when the disciplinary fidelity uncertainty is modeled using probabilistic techniques. It is felt conceptual design decisions arising out of such a methodology are more realistic compared to that from a purely low fidelity based deterministic design process.

Acknowledgements
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References

12. Vikram Sarabhai Space Center, Thiruvananthapuram, India *User’s Manual for PARAS-3D*.
Figure 1. Flow chart for Configuration Design of Generic Air-Breathing Hypersonic Vehicle

Figure 2. Parameterization of a Generic Air-Breathing Hypersonic Vehicle
Figure 3. Cross-sectional variation of pressure from High fidelity simulation

Figure 4. Input Uncertainty Model for Mass flow capture of air; Weibull Distribution Parameters: $\beta = 6.24$, $\alpha = 0.16$, $\theta = 1.00$

Figure 5. Input Histogram for Monte-Carlo Simulations

Figure 6. Typical Output Histogram from Monte-Carlo Simulation

Figure 7. Typical Cumulative Distribution for System metric
**Figure 8. Typical Reliability for System metric**

**Figure 9. Cross-Validation Diagnostic of DACE fit for System metric**

**Table 1. Design variables and Trade-Offs**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performance parameters that are influenced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nose tip divergence angle</td>
<td>Skin friction drag and volume available</td>
</tr>
<tr>
<td>Forebody compression angles</td>
<td>Fore-body length, height and volume available</td>
</tr>
<tr>
<td></td>
<td>Intake entry Mach number and static pressure</td>
</tr>
<tr>
<td>Wing cant angle</td>
<td>Drag, trim angle of attack</td>
</tr>
<tr>
<td>Wing and Tail planform factors</td>
<td>Wing and tail aerodynamic force and moments</td>
</tr>
<tr>
<td>Cruise altitude</td>
<td>Mass flow capture of air, drag, trim characteristics</td>
</tr>
</tbody>
</table>

**Table 2. Typical comparison of Aerodynamic characteristics between engineering method and CFD : M=6.5; α = 6°**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Tangent Cone / Wedge Method CN</th>
<th>Xcp/d</th>
<th>CA</th>
<th>CFD – Euler CN</th>
<th>Xcp/d</th>
<th>CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Alone</td>
<td>1.27</td>
<td>2.86</td>
<td>0.28</td>
<td>1.16</td>
<td>2.60</td>
<td>0.23</td>
</tr>
<tr>
<td>Body+Wing+Tail</td>
<td>2.03</td>
<td>3.91</td>
<td>0.43</td>
<td>1.66</td>
<td>3.51</td>
<td>0.35</td>
</tr>
</tbody>
</table>

CN and CA are normal and axial force coefficients d = 1m

**Table 3. Typical comparisons of captured mass flow of air estimated using CFD and oblique shock theory**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Fore-body Compression angles $\theta_1, \theta_2, \theta_3$</th>
<th>Euler CFD $m_a$, kg/s</th>
<th>Oblique Shock theory $m_a$, kg/s</th>
<th>Correlation Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.671, 2.491, 1.748</td>
<td>7.55</td>
<td>8.494</td>
<td>1.125</td>
</tr>
<tr>
<td>2</td>
<td>5.780, 5.328, 5.238</td>
<td>13.117</td>
<td>15.909</td>
<td>1.213</td>
</tr>
<tr>
<td>4</td>
<td>3.665, 2.712, 1.246</td>
<td>8.088</td>
<td>9.376</td>
<td>1.159</td>
</tr>
</tbody>
</table>
Table 4. Optimization Problem Statement
Minimize $F : -(\frac{Th_{deliv}}{AF} - 1)$
Subject to
\begin{align*}
G1: & \quad M_i / 4.2 - 1 \leq 0 \\
G2: & \quad \delta_{\text{min}} / 20.0 - 1 \leq 0 \\
G3: & \quad TOGW / 1200.0 - 1 \leq 0 \\
G4: & \quad L / 7.0 - 1 \leq 0 \\
G5: & \quad H / 0.8 - 1 \leq 0 \\
\end{align*}
Side Constraints
\begin{align*}
1.0 \leq \theta_1 \leq 6.0 \quad ; \quad 0.0 \leq \theta_w \leq 6.0 \quad ; \quad 30.0 \leq H_{cr} \leq 35.0 \\
1.0 \leq \theta_2 \leq 6.0 \quad ; \quad 0.8 \leq w_{pl} \leq 1.0 \\
1.0 \leq \theta_3 \leq 6.0 \quad ; \quad 0.8 \leq t_{pl} \leq 1.1 \\
\end{align*}

Table 5. Optimization Results

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Design Variables and Objective Function</th>
<th>Design-Exercise 1 Low Fidelity Based</th>
<th>Design-Exercise 2 Probability Based</th>
<th>Design-Exercise 3 High Fidelity Based</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\theta_1$, degrees</td>
<td>5.337</td>
<td>5.343</td>
<td>5.576</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_2$, degrees</td>
<td>4.034</td>
<td>3.504</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>$\theta_3$, degrees</td>
<td>3.448</td>
<td>3.972</td>
<td>6.000</td>
</tr>
<tr>
<td>4</td>
<td>$\theta_w$, degrees</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
</tr>
<tr>
<td>5</td>
<td>$w_{pl}$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>6</td>
<td>$t_{pl}$</td>
<td>0.870</td>
<td>1.026</td>
<td>0.800</td>
</tr>
<tr>
<td>7</td>
<td>$H_{cr}$, km</td>
<td>30.000</td>
<td>30.050</td>
<td>30.060</td>
</tr>
<tr>
<td>8</td>
<td>$F^*$</td>
<td>0.785</td>
<td>0.509</td>
<td>0.590</td>
</tr>
<tr>
<td>9</td>
<td>$G1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>$G2$</td>
<td>-0.33</td>
<td>-0.409</td>
<td>-0.29</td>
</tr>
<tr>
<td>11</td>
<td>$G3$</td>
<td>-0.143</td>
<td>-0.14</td>
<td>-0.153</td>
</tr>
<tr>
<td>12</td>
<td>$G4$</td>
<td>0</td>
<td>0</td>
<td>-0.021</td>
</tr>
<tr>
<td>13</td>
<td>$G5$</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.075</td>
</tr>
</tbody>
</table>

Note: $F^*$ computed based on high fidelity simulation for disciplinary metric corresponding to design variable settings of Design-Exercise 2 is 0.533